p-adic formulation of non-local gravity

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Outline

- **Problems to be addressed**
- **Strings, SFT, p-adic strings and all of that**
- $\mathcal{F}(\Box)$ **physics and Ostrogradski instability**
- **Non-local gravity as GR completion**
- p**-adic like reformulation of the non-local gravity**
- **Conclusion and open questions**

Problems

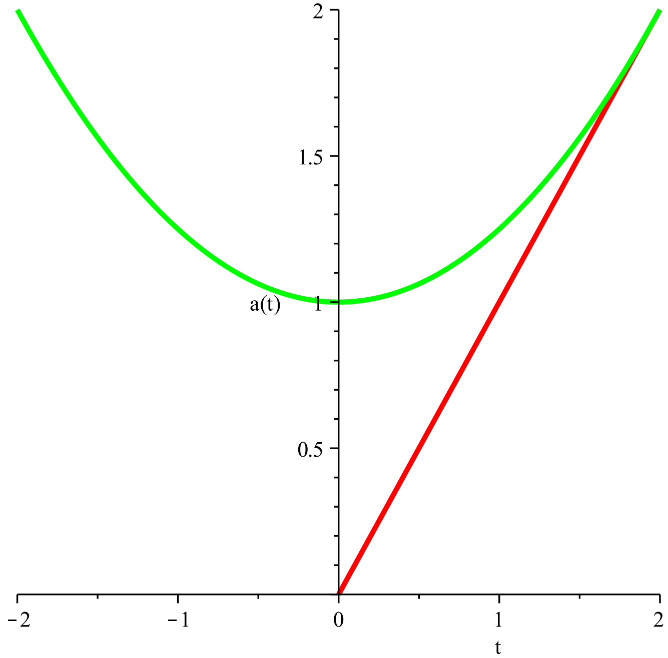
- **Cosmology and gravity do require graceful resolutions of singularities**
- **The major issues are initial singularity (Big Bang) and black hole singularities**
- **The initial singularity problem is the problem with most cosmological solutions because they hit a singularity approaching the time when Universe had begun**
- **Standard ways to avoid an initial singularity meet the more serious problem of ghosts**
- **These ghosts are related to higher derivatives in for example $f(R)$ models which may feature non-singular bouncing solutions**
- **Is there a way around?**

Initial singularity in pictures

- **Spatially flat FRW metric in 4 dimensions**

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & a(t)^2 & & \\ & & a(t)^2 & \\ & & & a(t)^2 \end{pmatrix}$$

- **Singular (Big Bang) vs non-singular (bounce)**



Strings

- **Strings step down from point-like objects and due to the presence of infinite dimensional conformal symmetry group in 2 dimensions feature a number of interesting properties**

- **SFT is the non-perturbative description of strings. UV completeness is one of the successes of SFT**
  Witten,Aref'eva,Medvedev,Zubarev,Preitschopf,Thorn,Yost,...

- **p-adic string theory is an effective theory capturing properties of the SFT scattering amplitudes**

$$L = \frac{m^D}{g_p^2}\frac{p^2}{p-1}\left(-\frac{1}{2}\varphi p^{-\frac{\Box}{2m^2}}\varphi + \frac{1}{p+1}\varphi^{p+1}\right)$$

Vladimirov,Volovich,Zelenov,Dragovich,Khrennikov,Brekke,Freund,Olson, Witten,...

$\mathcal{F}(\Box)$ physics

The Lagrangian to understand is

$$S = \int d^D x \left(\frac{1}{2} \varphi \mathcal{F}(\Box) \varphi - \lambda v(\varphi) + \ldots \right)$$

$\mathcal{F}(\Box) = \sum_{n \geq 0} f_n \Box^n$, **i.e. it is an analytic function.**

Canonical physics has $\mathcal{F}(\Box) = \Box - m^2$, **i.e.** $L = \frac{1}{2}\varphi\Box\varphi - \frac{m^2}{2}\varphi^2$

Ostrogradski statement says that higher (more than 2**) derivative in the Lagrangian can be eliminated by a field redefinition but new degrees of freedom will be ghosts (wrong kinetic term sign) or tachyons (wrong mass term sign), or both.**

$\mathcal{F}(\Box) = \Box^2 + f_1\Box - m^2$ **is an example.**

$\mathcal{F}(\Box)$ physics (continued)

A word "finite" was not explicitly in the cited statement and it appears to be crucial.

So, lets go "infinite", which implicitly means create a non-local Lagrangian.

Two examples are in order

$$\mathcal{F}(\Box) = (\Box - m^2)e^{-\beta\Box}$$
$$\mathcal{F}(\Box) = e^{-\beta\Box} - m^2$$

Roots of an equation

$$\mathcal{F}(\sigma) = 0$$

is the key to understand the physics here.

The *ghost-free* condition requires no more than one root σ exists.

Non-local gravity

The following Lagrangian describes the modification of gravity expected from the closed String Field Theory.

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2R}{2}+\frac{\lambda}{2}R\mathcal{F}(\Box)R-\Lambda+\dots\right)\ \textbf{notice}\ M_P^2=\frac{1}{8\pi G_N}$$

Biswas,Koivisto,Mazumdar,Siegel,Dragovich,Vernov,AK,...

One of the equations of motion (trace) is

$$\lambda\sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}\left(\partial_\mu\Box^l R\partial^\mu\Box^{n-1-l}R+2\Box^l R\Box^{n-l}R\right)+6\lambda\Box\mathcal{F}(\Box)R-M_P^2R=-4\Lambda$$

The ghost-free condition in Minkowski space ($\Lambda=0$) requires that the following equation

$$6\lambda\sigma\mathcal{F}(\sigma)-M_P^2=0$$

has no more than one root σ.

Non-local gravity (continued)

It is really not obvious but the above mentioned equation has the following explicit analytic solutions

- **We need** $\mathcal{F}'(\beta) = 0$ **and some radiation**

$$a = a_0 \cosh(\beta t) \;\Rightarrow\; H = \beta \tanh(\beta t)$$

- **We need** $\mathcal{F}'(\beta) = 0$ **and NO radiation**

$$a = a_0 \exp\left(\frac{\beta}{2} t^2\right) \;\Rightarrow\; H = \beta t$$

- **We need some more special condition on** $\mathcal{F}(\sigma)$ **and NO radiation**

$$a = a_0 t^p \;\Rightarrow\; H = \frac{p}{t}$$

The two first solutions are the manifestly non-singular bouncing solutions moreover with a de Sitter late time asymptotic for the first one

p-adic reformulation of the non-local gravity

The previous action is equivalent to the following one

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2}\left(1+\frac{2}{M_P^2}\psi\right)-\frac{1}{2\lambda}\psi\frac{1}{\mathcal{F}(\Box)}\psi+\dots\right)$$

An important property here is the non-minimal coupling of a scalar field to gravity.

The conformal transform $\left(1+\frac{2}{M_P^2}\psi\right)^2 g_{\mu\nu}=\bar{g}_{\mu\nu}$ **allows us to decouple the gravity and the scalar field even more**

$$S=\int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R}-\frac{M_P^2}{2}\frac{6}{(M_P^2+2\psi)^2}\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi-\frac{M_P^4}{2\lambda(M_P^2+2\psi)^2}\psi\mathcal{G}(\mathcal{P})\psi\right)$$

Here

$$\mathcal{G}(\sigma)=\frac{1}{\mathcal{F}(\sigma)}\text{ and }\mathcal{P}=\left(1+\frac{2}{M_P^2}\psi\right)\Box_{\bar{g}}-\frac{2}{M_P^2}\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu$$

Conclusions and open questions

- p-adic string theory triggers the development of non-local field theories
- Non-local generalization of Einstein's gravity is presented with the aim at resolving the initial singularity problem
- We were able to find out explicit analytic solutions representing the non-singular bounce
- p-adic reformulation of the non-local gravity is presented and important properties are discussed
- The current work in progress is to formulate a connection of the non-local gravity in its p-adic formulation with the so called galileon models
- Moreover there is a hope the latter formulation of the non-local gravity will allow us to find explicitly new black objects

Thank you for listening!